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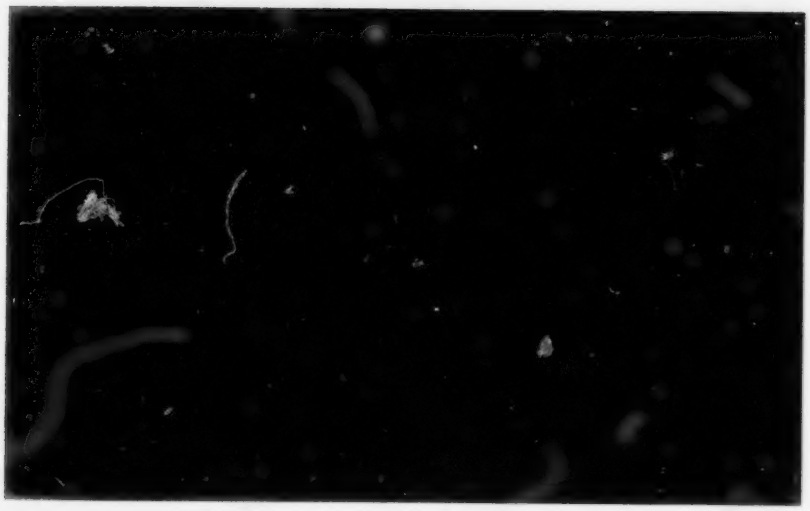

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A Decision Support System for Physical Distribution Planning

by

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Abstract

A manager planning a distribution network faces a multiple-vehicle routing problem. The first step is to assign customers to vehicles. One way to do this is to partition a given territory into zones assigned to each vehicle. This paper presents a decision support system that enables the distribution manager to interactively design the zones at the screen using his own judgement as well as suggestions made by the system. The latter are based on analytic formulae that determine the optimal shape of a zone. This helps the planner design the zones while taking into account the speed of the vehicle, the density of the demand, the location of the zone, and a maximal duration or a maximum number of stops in a route, as well as his own knowledge of the territory.

Keywords: Decision, design, distribution, planning.

Résumé

Dans la planification d'un réseau de distribution, on fait face à un problème de routage d'un ensemble de véhicules. Une façon d'aborder ce problème consiste à subdiviser le territoire à couvrir en zones affectées aux véhicules. Cet article présente un système d'aide à la décision qui permet au gérant de la distribution de désigner interactivement les zones en utilisant son jugement de même que des suggestions émises par le système. Ces dernières sont basées sur des formules analytiques qui déterminent la forme optimale d'une zone. Ceci permet de tenir compte de la vitesse des véhicules, de la densité de la demande, de la localisation de la zone, et de la durée ou du nombre maximum d'arrêts d'une tournée de même que de la connaissance que le planificateur a du territoire.

Mots-clés: décision, design, distribution, planification.

1. Introduction

Logistics is concerned with the movement of goods and people in space and time. In commerce and industry a logistics system is often described as one enabling the flow of materials and goods from the suppliers to the factory, then to warehouses or depots and finally to the customers. Physical distribution relates to the movement of goods from the factory to the customers. In this article, the definition of physical distribution is restrained to the movement of goods from one or many depots to the customers.

When the goods are shipped via a fleet of delivery vehicles, the distribution manager faces a difficult routing problem: optimal multiple-vehicle routing is a complex problem to solve because of its highly combinatoric nature. An efficient way to tackle the problem is to first partition the customers into clusters and then to solve in each cluster a one-vehicle routing problem using an optimal or heuristic method. This approach is known in the literature as "clustering first / routing second".

In urban areas, the routing problem is much more difficult to solve because accuracy of the needed data (mainly the distances or times between customers) is tedious to obtain and keep up to date. Weather, traffic, state of the road network or hazards, for instance, may substantially alter the validity of the data. Hence, it is certainly more efficient, once the clustering is done, to let the drivers do the actual routing. They usually have a very good knowledge of their delivery zone. Though not optimal, those routes could be expected to be relatively good.

The distribution manager is thus left with only the clustering problem. One way

to do this is to partition the whole area into zones, each zone being serviced by a single vehicle.

This article describes a decision support system (DSS) that enables the manager to interactively design the delivery zones on a displayed map. The objective is to allow the planner to incorporate his knowledge of the region into the design process while being guided by analytical tools. This kind of knowledge, which includes all sorts of geographical and road network constraints, would otherwise be difficult to model.

The system described herein goes further than merely presenting useful information to the manager. It provides analytical support for decision making in an interactive environment. It does not automate the decision process nor impose solutions. To the contrary, the user maintains control at all times and sees his range of capability extended as the system presents him optimal suggestions for the next decision in a step by step process. At any time, the user may discard any of those suggestions preferring to use his own knowledge. Although the underlying design problem is sufficiently structured to justify computer and analytic aid, the user's judgement remains essential.

The system could also be used at a strategic level to study the impact of the growth of the demand, to evaluate different depot locations, or to study the impact of using additional depots, changing the size of vehicles, modifying the duration of the routes (through union agreement) etc.

In Section 2 of the paper we review the analytic formulae upon which the DSS relies. The formulae give the expected length of route as functions of the number of

points to visit and the area covered as well as the optimal dimensions of the area. Section 3 describes the DSS. In Section 4, we illustrate the use of the DSS for the planning of the pick-up and delivery zones for a courier company.

2. Analytic formulae for estimating route length

In order to determine the optimal shape and dimensions of a delivery zone, it is necessary to estimate the length of a tour in relation to the area and shape of the zone and to the the number of points to be visited. Larson and Odoni (1981) present an approximation formula for the length L_n^* of an optimal tour of n points randomly and uniformly distributed over a zone of area A (the area being relatively compact and convex):

$$L_n^* = k\sqrt{An} \tag{1}$$

This formula has been derived from the asymptotic formula of Beardwood et al. (1959). The k value has been estimated at 0.765 for the Euclidean metric by Stein (1978) and at 0.97 for the Manhattan metric by Jaillet and Odoni (1988).

Equation (1) does not, however, relate the length of the tour to the shape of the zone nor define the way to obtain the optimal tour. To circumvent this shortcoming, Daganzo (1984) proposes a suboptimal routing strategy: the zone is covered with a strip of width w and the points are visited in longitudinal order within the strip (fig. 1).

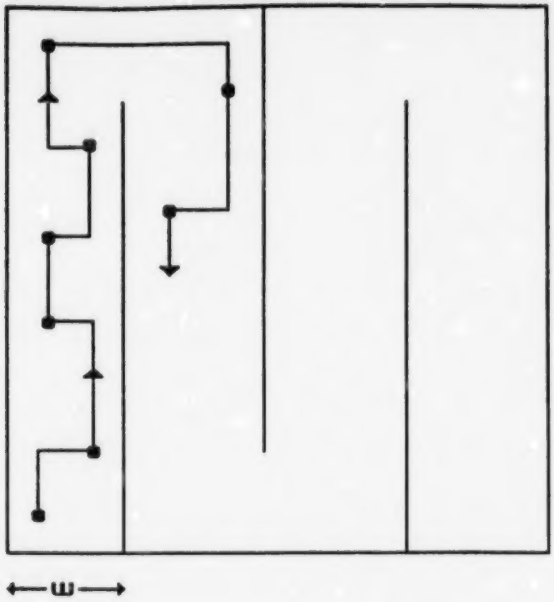

Figure 1 : The routing strategy with a strip of width w

The optimal width of the strip with the Manhattan metric is $w^* = \sqrt{3/\delta}$ ($\sqrt{2.95/\delta}$ with Euclidean metric) where δ is the density of clients *(n/A)*. The distance traveled using this "strip" strategy (with w^*) is estimated at $L_n^* = 1,15\sqrt{An}$ (0,97 in the Euclidean case). This strategy, though not optimal, greatly facilitates the routing. It is easy to use manually or to implement on a computer. It would probably give routes with a length close to what drivers actually achieve in practice. Moreover, Newell and Daganzo (1986) have shown how easily those solutions may be improved to approach optimal solutions.

Newell and Daganzo use the "strip" strategy to analyse the zone design under vehicle capacity constraint and when each vehicle departs from and returns to a depot. They show that in a multiple-vehicle routing problem, each delivery zone should have in a ring radial network a rectangular shape elongated toward the depot. Each zone should be approximately $2w^*$ wide so that each vehicle does only one round trip in

it. Starting and ending the delivery tour of a zone on the side nearest to the depot minimizes the line-haul distance (fig. 2).

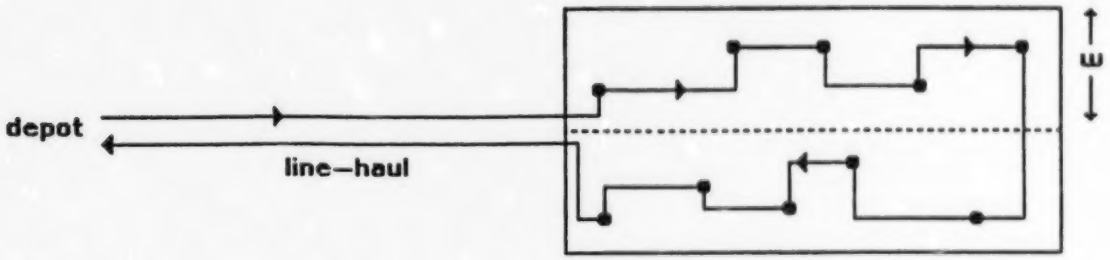

Figure 2 : A single vehicle delivery zone

Let r be the distance from the depot to the center of the zone and C the maximum number of points to be visited by a single vehicle. The optimal width and length of the zone are given by

$$w^* = \sqrt{3/2\delta} \tag{2}$$

$$\ell^* = C/\sqrt{6\delta} \tag{3}$$

It should be noted that those values are independent of the distance between the zone and the depot. Using w^* and ℓ^*, it is straightforward to derive the total traveled distance:

$$L_n^* = 2r + C\sqrt{2/3\delta} \tag{4}$$

Langevin and Soumis (1989) consider the zone design problem under a route duration constraint. Using the same approach as Newell and Daganzo (1986), (Figure

2), they demonstrate that the optimal width and length are given by

$$w^* = \sqrt{3/2\delta}$$

$$\ell^* = \frac{Tv-2r}{2+\sqrt{6\delta}.\ rv} \tag{5}$$

where v is the vehicle speed, T the maximal duration of the tour and τ the handling time at each delivery point. With the optimal shape, the duration of the tour equals the maximum duration T, meaning that the vehicle is fully utilised. The total traveled distance is:

$$L_n^* = 2r + A\sqrt{2\delta/3} \tag{6}$$

where $A = 2w^*\ell^*$. Equation (6) is equivalent to equation (4). The duration of the trip is:

$$D^* = \frac{L_n^*}{v} + \delta A\tau \tag{7}$$

3. The Decision Support System

3.1 The method

To determine the optimal shape and area of the delivery zone for each vehicle, Langevin and Soumis (1989) like Newell and Danganzo (1986) work on a ring radial network centered around the depot. With the density of clients defined as a function of the distance to the depot, they show how to construct the optimal zones starting at the periphery and gradually closing in to the center. While being interesting from a theoretical point of view, this approach is not readily applicable to real life situations. The following are some of the many reasons why: urban areas are not usually circular

with a perfect ring radial network; the depot may be located anywhere out of the geographical center; the density may not vary with the distance to the depot.

The theory may nevertheless be used as a guideline in the construction of the delivery zones to avoid gross suboptimal choices. However, the control should remain in the hands of the planner at all times.

The aim is to minimize the total traveled distance for all the vehicles subject to either a maximum number of clients or a maximum length of time per vehicle (as chosen by the user).

Starting with a large set of small geographic sections of the map called polygons, the method involves in choosing a first polygon, then aggregating adjacent ones , one at a time, to form the first delivery zone. The creation of the next zones follows the same procedure. To help the user choose the next polygon to add, the system displays the number of clients serviced and the duration of the trip for the polygons already assembled. This assists the user in respecting the capacity or time constraint. The system also superimposes on the already chosen polygons the optimal rectangular shape of the delivery zone as illustrated in figure 3.

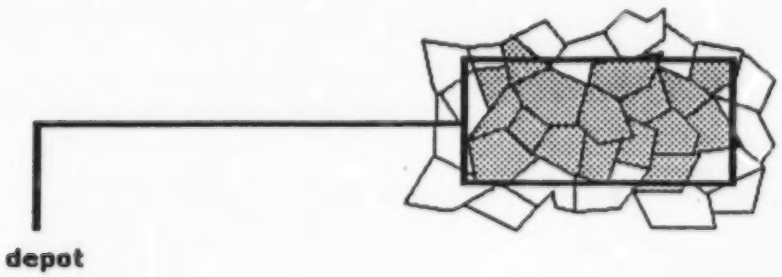

Figure 3 : Optimal shape of delivery zone over polygons

With this approach, the user may more readily see what neighboring polygon would be good to aggregate according to its shape, area and position.

The implementation of the practical approach requires numerical information to be fed into the following formulae which were introduced in Section 2.

1) $L_n^* = 2r + A\sqrt{2\delta/3}$ to determine the optimal travel distance within the current cluster of polygons plus the line-haul distance from and to the depot.
2) $n = A\delta$ to determine the number of clients within a polygon (or a cluster of them).
3) $D^* = \frac{L_n^*}{v} + \delta A\tau$ to determine the duration of a trip within a polygon (or a cluster of them).
4) $w^* = \sqrt{3/2\delta}$ to determine the half width of an optimal rectangular zone with either a capacity or a time constraint.
5) $\ell^* = c/\sqrt{6\delta}$ or $\ell^* = \frac{Tv-2r}{2+\sqrt{6\delta}.\,rv}$ to determine the length of an optimal zone with a capacity or time constraint.

The following data are required to start the process; their values may be modified at any time and the system will adjust itself accordingly.

1) The position of the depot (to calculate the value of "r"). It is entered by selecting a point on the map.
2) The constraint, be it the maximum number of clients or the maximum duration of the travel.
3) The speed of the vehicule.

4) The handling time at each client's location.

5) The road grid axis (assuming a Manhattan metric).

6) The density of clients δ_i for each polygon.

After retrieving a map, the system automatically calculates the centroid $(x_i; y_i)$ and the area a_i of each polygon. As each zone is gradually created by adjoining one polygon at a time to those previously selected, the area, density and centroid for the current zone are calculated as follows:

1) The area of a zone : $a = \sum a_i$

2) The density of a zone : $\delta = \frac{\sum \delta_i a_i}{\sum a_i}$

3) The coordinates of the centroid of a zone: $x = \frac{\sum x_i \delta_i a_i}{\sum \delta_i a_i} \quad y = \frac{\sum y_i \delta_i a_i}{\sum \delta_i a_i}$

It should be noted that the width $2w^* = 2\sqrt{3/2\delta}$ of the optimal rectangular sector is solely a function of the density of clients. It is totally independent of other variables.

As the superimposed rectangle is recalculated after each addition of a polygon, the user may at will add a polygon, delete it, add another and see instantly the overall effect on the duration of the tour, the number of clients or the area covered. This gives him both a graphical and quantitative view of each choice possible at each step of the process. The user will therefore be able to create solutions respecting any type of geographical or human constraint deemed important without compromising too much on the optimum.

3.2 The prototype

In this section, the use of the software implementation of the DSS is described. The software is called "Prototype Plani-Zone".

The windows and menu first offered to the user appear in figure 4. The largest window will contain the map of the area considered and the user may select any command in the menu by clicking with the mouse.

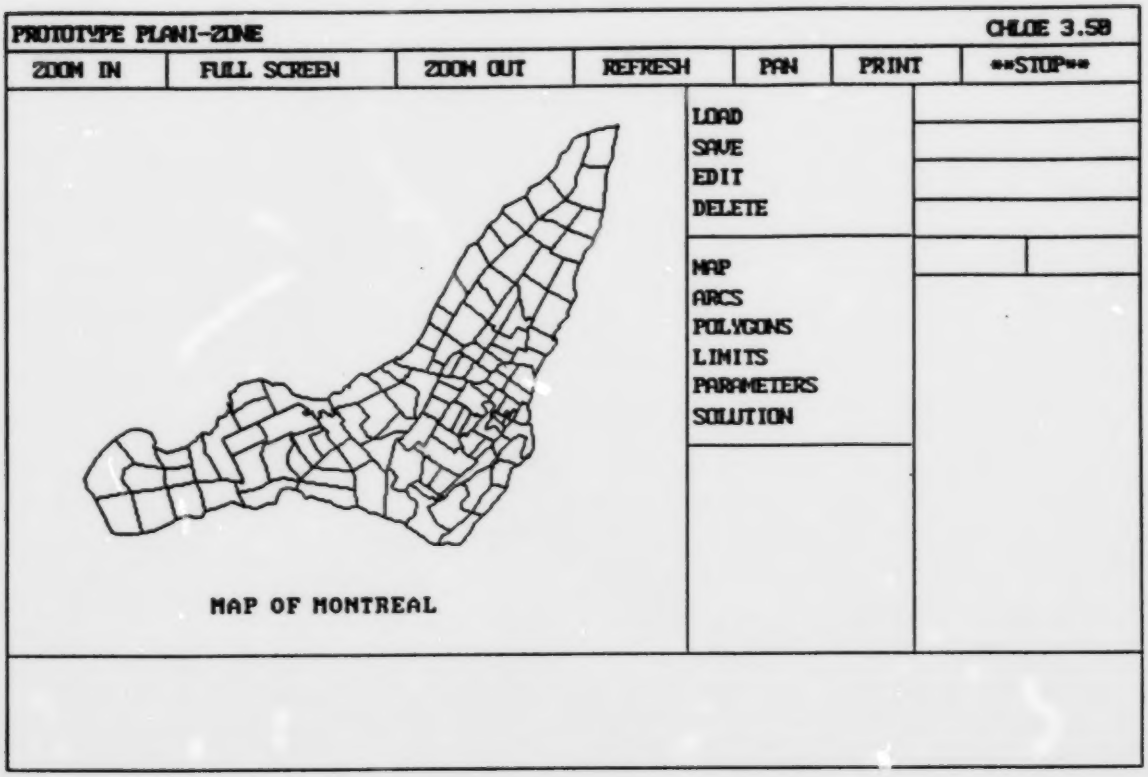

Figure 4 : The first screen with the map of Montreal

A work session is composed of the following steps:

1) Retrieve a map and display its polygons.

2) Enter or modify the parameters:

   — type of constraint;

   — polygon densities;

   — vehicle speed;

   — depot location;

   — handling time;

   — orientation of road grid.

3) Create a delivery zone:

   A) Sequentially select (with the mouse) a number of polygons.

   B) Add or delete polygons until satisfied with the current zone. As each polygon is added or deleted, the system displays the following information to guide the user (see figure 5):

      - The optimal shape of the delivery zone superimposed on the current zone. It is calculated as a function of the centroid of the current zone, its distance to the depot and the six values mentioned in section 3.1.
      - The maximum delivery time or the maximum number of clients specified by the constraint.
      - The total number of clients in the current zone.
      - The route duration for the current zone.

4) Repeat step 3) until all zones are created.

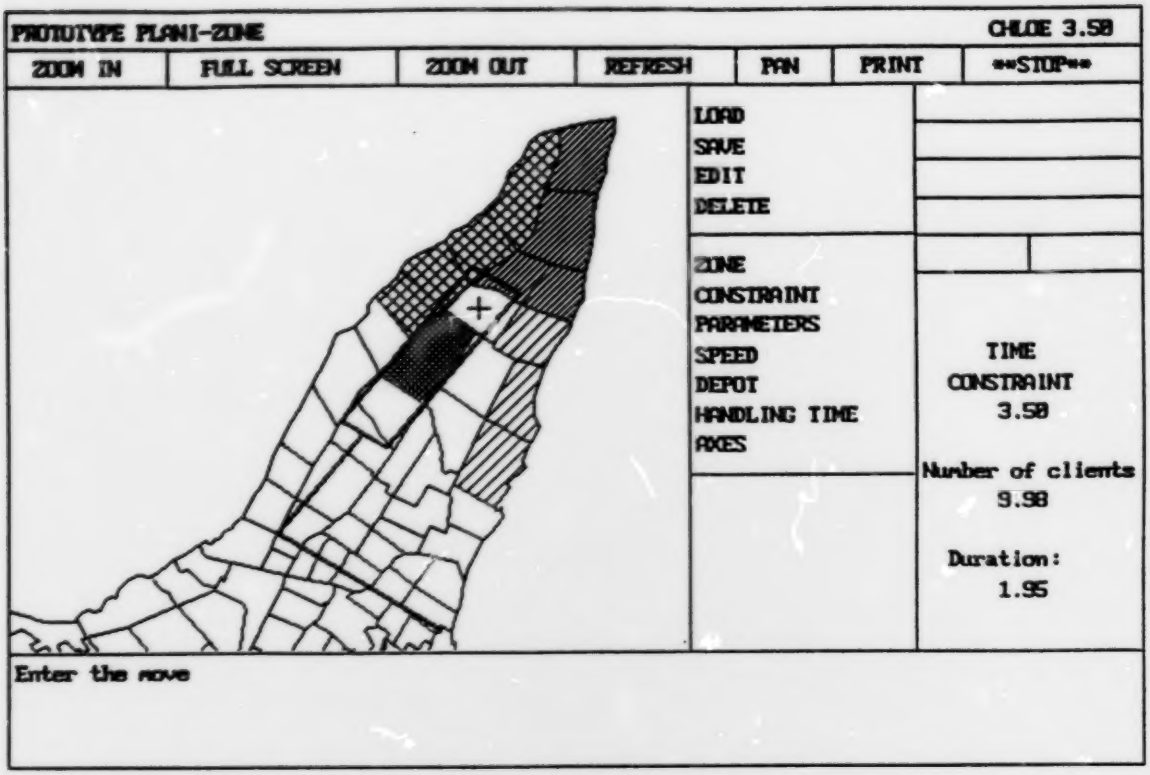

Figure 5 : Working on the map. Three zones have already been created and a fourth one is being assembled. At right, information about the current zone is displayed

To ensure maximal ease of use, the system provides facilities for editing and saving of all input data and solutions. This frees the user from the tedious task of entering the same information many times. Also provided are zoom-in, full-screen and displacement capabilities.

The prototype was constructed using a User Interface Management System called CHLOE (1990) developped by the MIAO group of Ecole Polytechnique de Montréal. It manages the various windows on the screen as well as the hierarchy of menus and sub-menus.

4. Using the DSS for replanning an actual delivery service

The DSS described in section 3 was actually used to plan the delivery zones for a large courier company in Montreal. The company offers a pick-up and delivery service: letters and small parcels are picked up on afternoons and delivered the next morning, after being sorted overnight at the depot. Each vehicle tour should last about 3.5 hours and, due to the small number of letters and parcels that a vehicle can deliver in that period, no capacity constraint is considered. Even though the demand may vary from day to day, it is advisable to maintain stable delivery zones so that each driver gets accustomed to his assigned zone. The polygons correspond to the first three characters of the postal codes. They are shown in figure 4. For each polygon the density of the demand is entered.

We started to assemble the first zones from the eastern end of the map (see figure 5). We then proceeded to the western end, and finished with the region surrounding the downtown area. As we moved from one zone to another, we adjusted the grid orientation using our knowledge of the region. We did not use the DSS to plan the zones in the downtown area because the model would not apply there. Also, a large part of the deliveries is made by foot in that sector.

Figure 6 shows a part of the complete solution. Each delivery zone is clearly identified and the system provides the expected duration and length of the tour, along with the estimated number of stops. With this system, it is easy for the manager to obtain a balanced partition of the work load between the drivers.

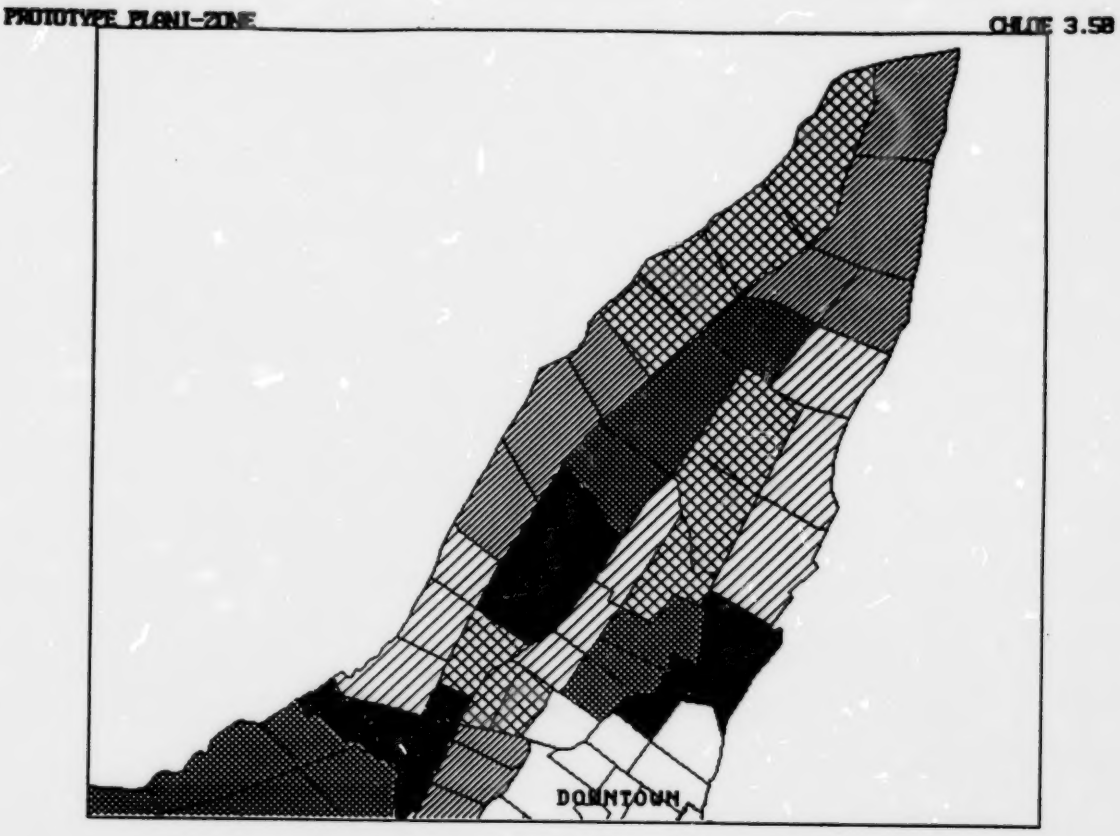

Figure 6 : The delivery zones

The system is user-friendly and any user can master it quickly. It is possible to take into account all sorts of geographical and human considerations in the zone design process. Guided by analytical tools, the manager maintains absolute control throughout the decision process.

5. Conclusion

We have presented a DSS for the design of delivery zones in physical distribution. The system is based on analytic formulae determining optimal shape and orientation of the zones. Those analytic tools combined with the planner's judgment allow him to interactively design the zones taking into account geographical and human constraints that are otherwise difficult to model. The system provides throughout the process essential informations about route duration and length. However, it does not automate the decision process.

The system is flexible and user-friendly. It can be used for planning at a strategic level. Impacts of any change in the distribution service can be easily evaluated (growth in demands, locations of depots, duration of routes, size of vehicles, etc.). Finally, the system presented herein shows that continuous approximations are valuable tools to solve complex logistics problems.

6. Acknowledgements

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